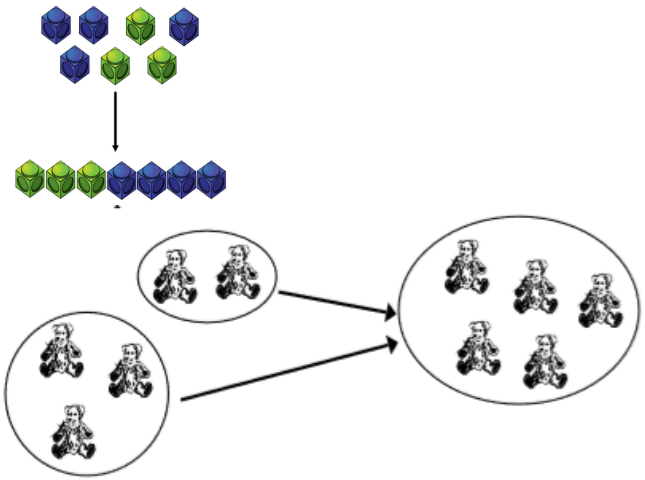
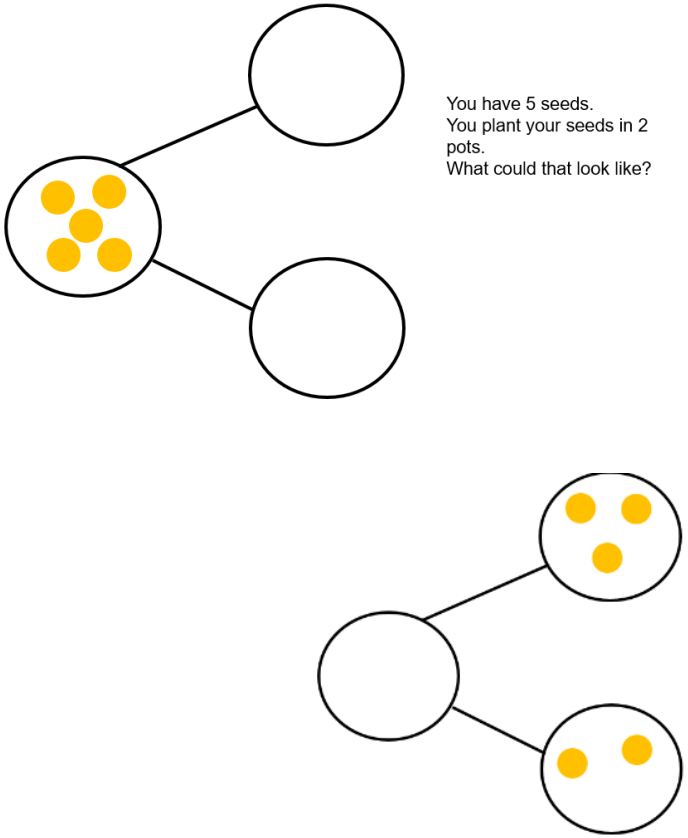
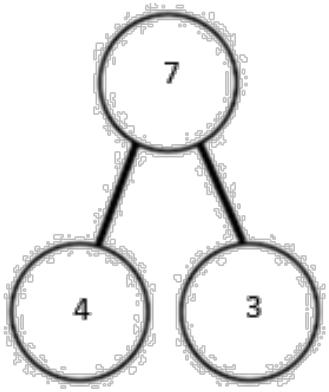
High Hazels Academy
The best in everyone™
Part of United Learning

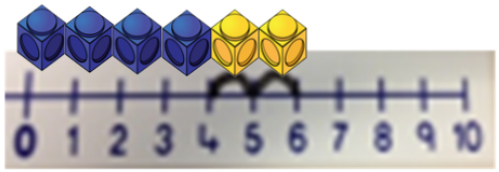
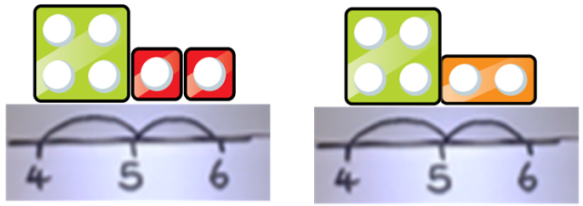
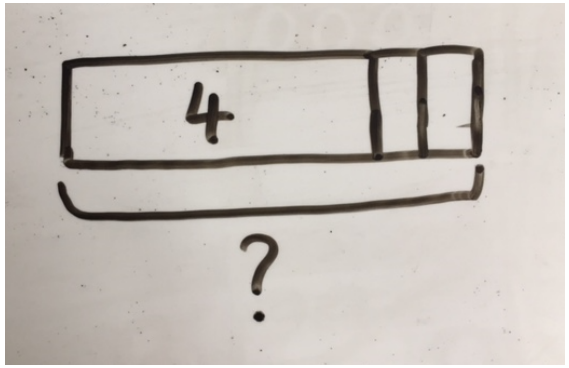
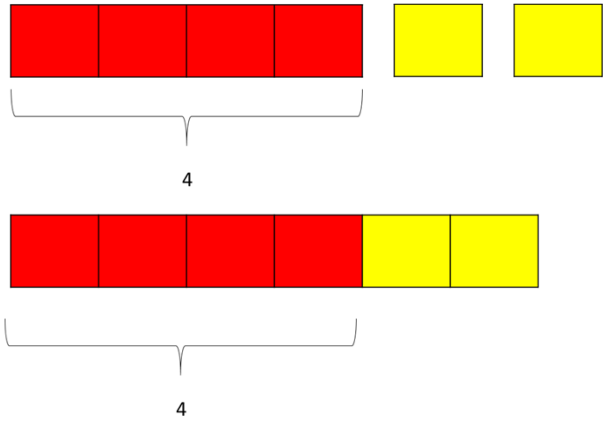
Calculation policy 2019/2020

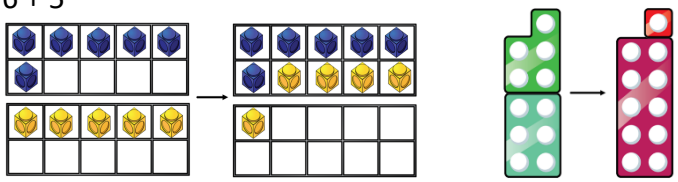
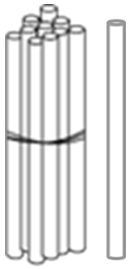
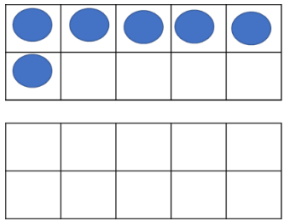
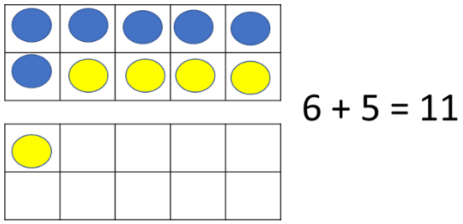
This document shows how the four operations are built up from KS1 to KS2. Various concrete, pictorial and abstract representations are shown. Remember, you should move between representations within a lesson, showing them alongside each other so children can see the connection between representations. This document also contains ideas for conceptual variation and intelligent practice to develop children's understanding further.

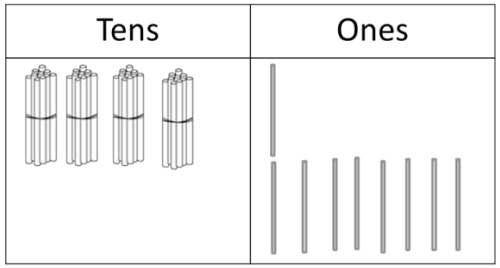
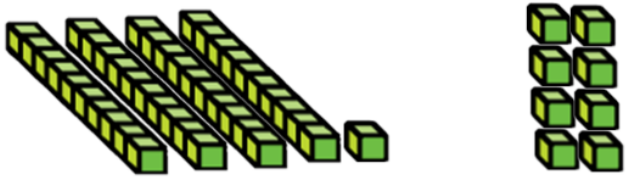
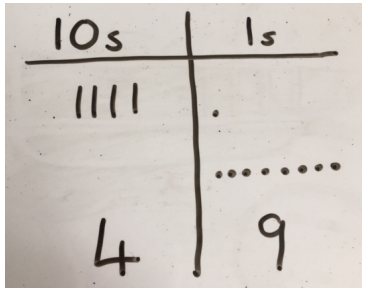
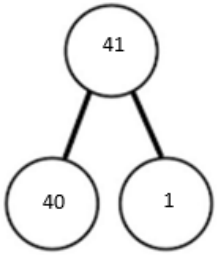
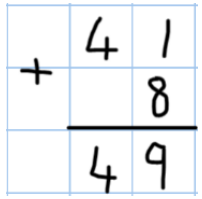
Calculation policy: addition

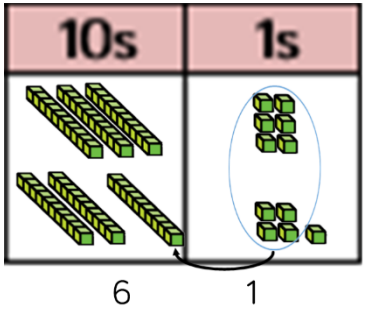
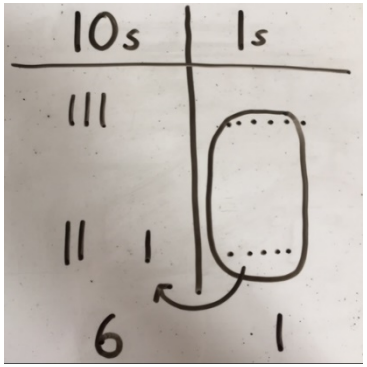
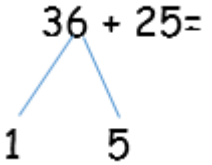
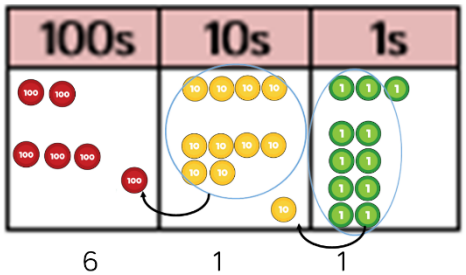
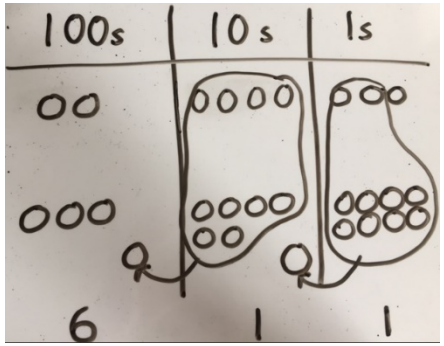
Key language: sum, total, parts and wholes, plus, add, altogether, more, ‘is equal to’ ‘is the same as’.

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use a range of resources e.g. eggs, shells, teddy bears, cars).  Children represent the real objects using counters as this bridges the gap between concrete and pictorial representations. Children physically partition the counters then recombine.	Children represent what they see pictorially.  	4 + 3 = 7 Four is a part, three is a part and the whole is seven. 

Concrete	Pictorial	Abstract
Counting on using a number track – children physically jump on a 'floor' track or using objects on a table number track. 0 1 2 3 4 5 6 7 8 9 10 **Counting on using number lines** with cubes or Numicon.   	Children draw a bar model which encourages them to count on, rather than count all.  Scaffold idea – the '4' can be represented as a block of four cubes but labelled with the value of 4. Encourage to count on from 4.  	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2  Possible 'counting on' not 'counting all' activity –roll a die, find the number on a numberline, roll again and then count on. Following a few tries, explore why it is effective to start with the larger value and count on the smaller. *Note – avoid numberlines in different colours, unless the pattern in the colours is the focus of the learning.*

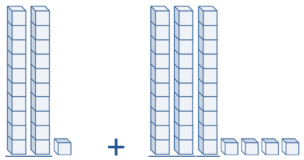
Concrete	Pictorial	Abstract
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5  Using straws/sticks, children count and regroup into bundles of 10. 	Children draw counters/cubes/dots onto a readymade, wipeable ten frames (could be inside poly-wallets, with ten frames partially completed).  6 + 5 =  6 + 5 = 11 Children draw the 5 counters they are adding. Focus on the 5 becoming a 4 and a 1 to enable use of number bonds.	Children to develop an understanding of equality e.g. 6 + □ = 11 11 = 6 + □ □ + 6 = 11 (commutative law – order of addition does not change the sum) 6 + 5 = 5 + □ Building towards… 6 + 5 = □ + 4 6 + 5 = □ + 3 6 + 5 = □ + 2 (intelligent practice – what is changing?)

Concrete	Pictorial	Abstract
TO + O using base 10 and straws/sticks. Continue to develop understanding of partitioning and place value. 41 + 8   Place these base 10 into a place value grid to reinforce place value.	Children represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 

Concrete	Pictorial	Abstract
TO + TO using base 10. Continue to develop understanding of partitioning and place value. *When there are 10 ones in the 1s column, we exchange for 1 ten.* 36 + 25  https://www.youtube.com/watch?v=HGk8F6rRpPg&index=2&list=PLQqF8sn28L9yEyXuHbQNSiQr6sCzqrFrR	Children represent the base 10 in a place value chart, circling when they make an exchange.  	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1
HTO + TO, HTO + HTO etc. using place value counters. *When there are 10 ones in the 1s column, we exchange for 1 ten. When there are 10 tens in the 10s column, we exchange for 1 hundred.*  	Children represent the counters in a place value chart, circling when they make an exchange.  	Formal method: 243 +368 611 1 1

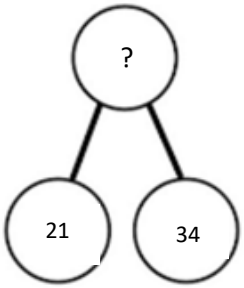
Concrete	Pictorial	Abstract

Conceptual variation: different ways to ask children to solve 21 + 34



Missing digit problems:

	Tens	Ones
	10 10	1
+	10 10 10	?
	?	5



?	
21	34

$$\begin{array}{r} 21 \\ +34 \\ \hline \quad\quad \end{array}$$

21 + 34 =

$\square$ = 21 + 34

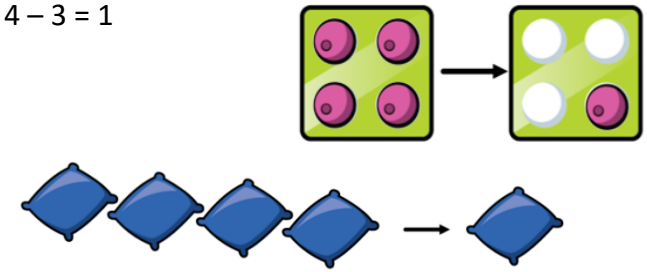
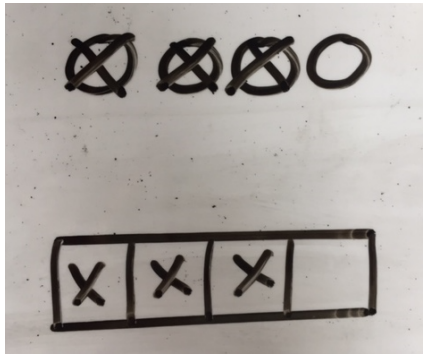
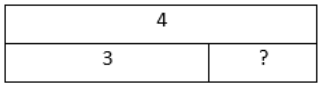
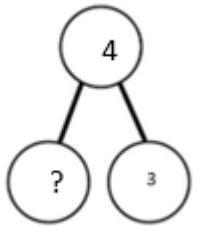
Calculate the sum of twenty-one and thirty-four.

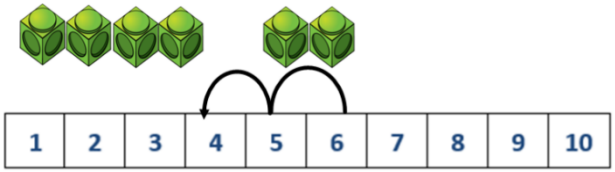
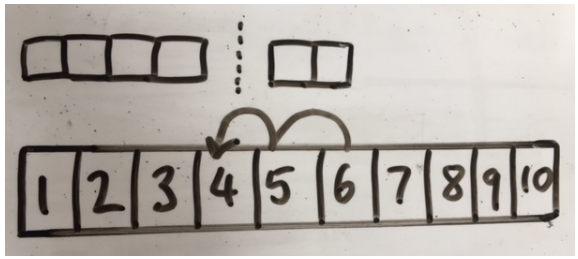
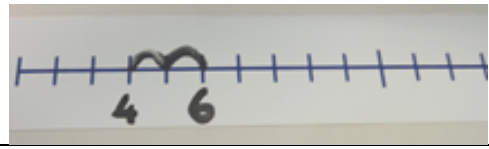
Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

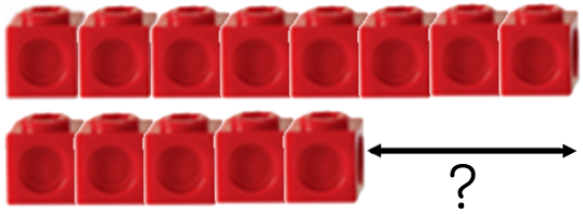
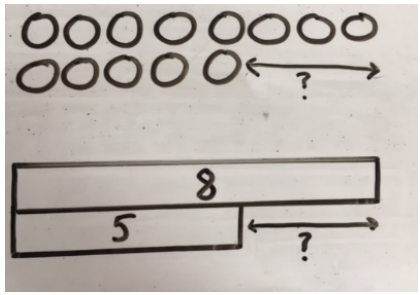
21 + 34 = 55. Prove it

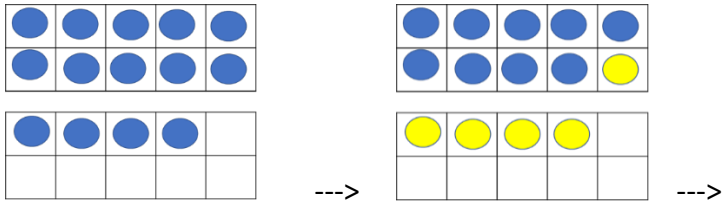
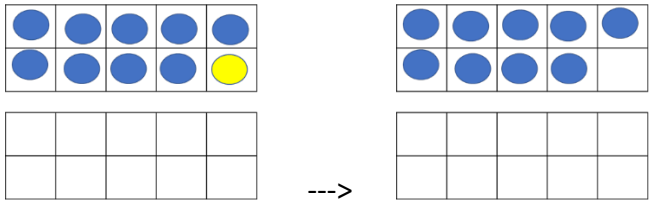
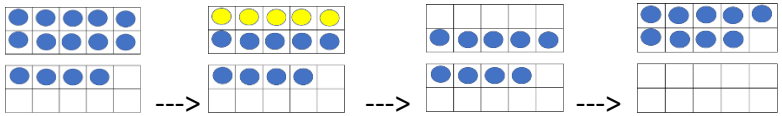
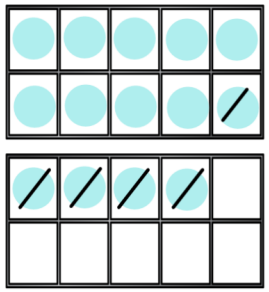
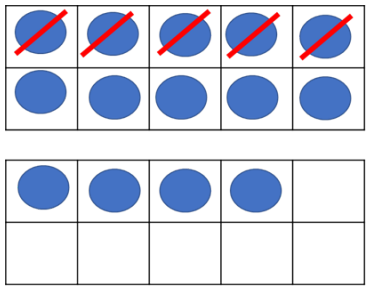
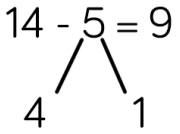
Calculation policy: subtraction

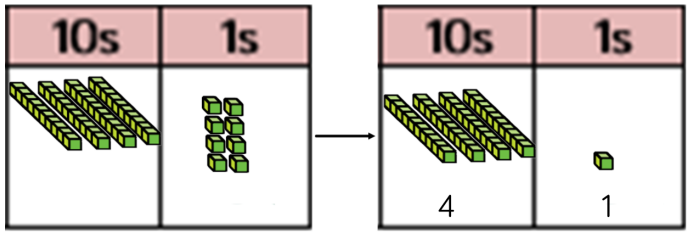
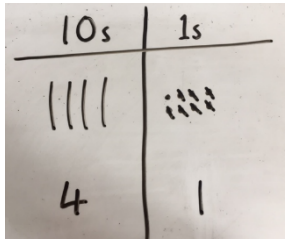
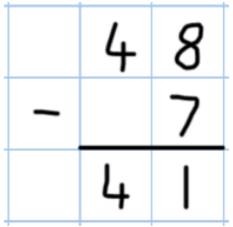
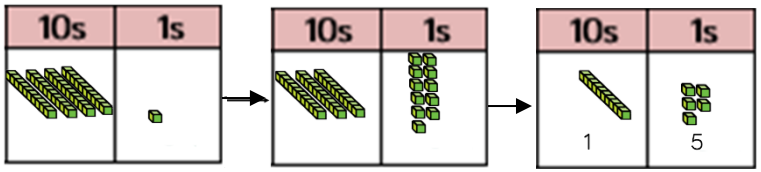
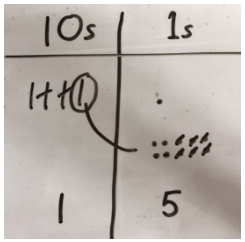
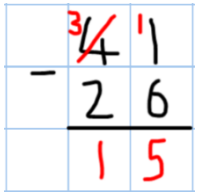
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

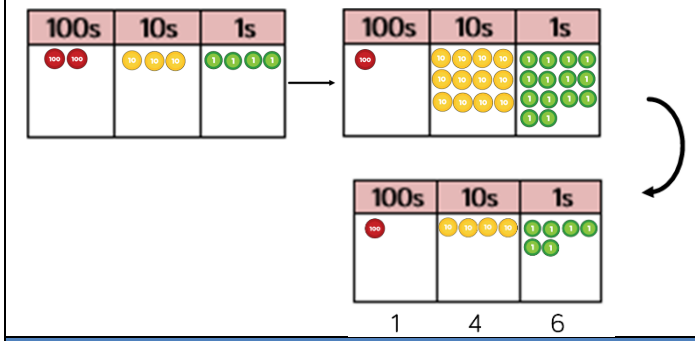
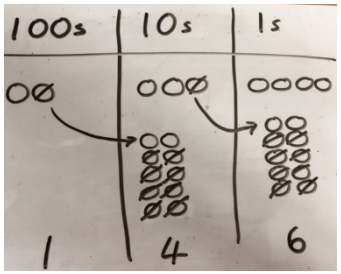
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1  Then use counters to represent the actual objects, as this bridges the gap from concrete to pictorial representations.	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.  https://cloud.swivl.com/v/df3d1d947cc0f8b5979b40c425f297c9	**4- 3 =** **= 4 – 3**  

Concrete	Pictorial	Abstract
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

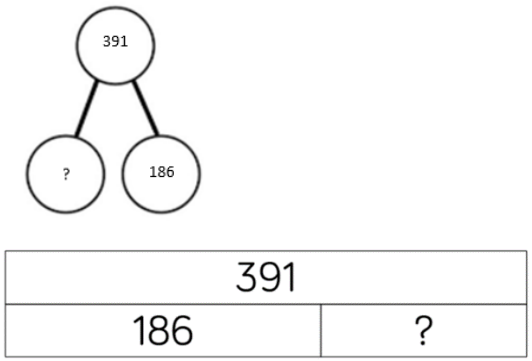
Concrete	Pictorial	Abstract
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children draw the concrete objects they have used or a bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ If 8-5=3, then 7 - 4 = 3 6 – 3 = 3 and so on Children to explore why … 9 - 6 = 8 – 5 9 - 6 = 7 – 4 9 - 6 = 6 – 3 … have the same difference. What about 10-? keeping the difference the same?

Concrete	Pictorial	Abstract
Applying knowledge of 10 on ten frames. 14 – 5  Turn these 5 counters over…  … remove the four… … and then the one. And as an alternative, take the 5 from the ten and recombine the rest of the counters into one ten frame:  https://cloud.swivl.com/v/901c5e28c274bd2649a36b742f603c12	Children represent the ten frame pictorially and discuss how they used their knowledge of ten to do the calculation. 14 – 5 =  And as an alternative… 	Children to show how they can make 10 by partitioning the subtrahend. (In the case below, 14 is the minuend and 5 is the subtrahend.)  14 – 4 = 10 10 – 1 = 9 And as an alternative… 10 – 5 + 4 = 9 The minuend is the quantity from which a value is subtracted. The Subtrahend is the value that is subtracted.

Concrete	Pictorial	Abstract
Column method using base 10. 48-7  Could also use straws/sticks N.B The 7 is not represented but taken away from the 48.	Children to represent the base 10 pictorially.  N.B Again, the 7 is not represented but taken away from the 48.	Column method or children could count back 7.  Could lay straws over the abstract calculation to link back to concrete materials: 
Column method using base 10 and having to exchange. 41 – 26  Could also use straws/sticks	Represent the base 10 pictorially, circling the exchange.  	Formal method: children must understand when they have exchanged the 10, they still have 41 because 41 = 30 + 11. 

Concrete	Pictorial	Abstract
Column method using place value counters. 234 – 88 First, exchange 1 ten into ten ones. Next, exchange 1 hundred into 10 tens/ 	Represent the place value counters pictorially, circling what has been exchanged. 	Formal method: children must understand what has happened when they have crossed out digits. $\begin{array}{r} {}^{2}\not{2}\,{}^{1}\not{3}4 \\ -\ 88 \\ \hline 6 \end{array}$

Conceptual variation; different ways to ask children to solve 391 - 186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

□ = 391 – 186

$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$

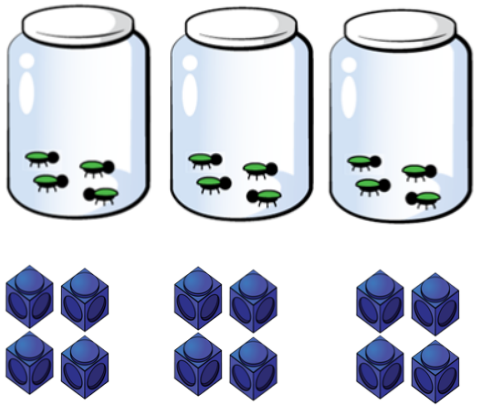
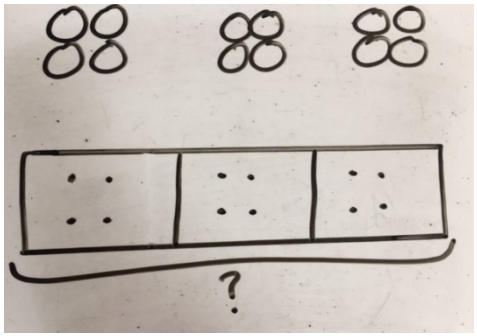
What is 186 less than 391?

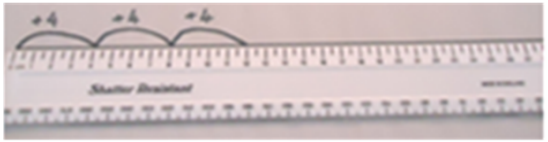
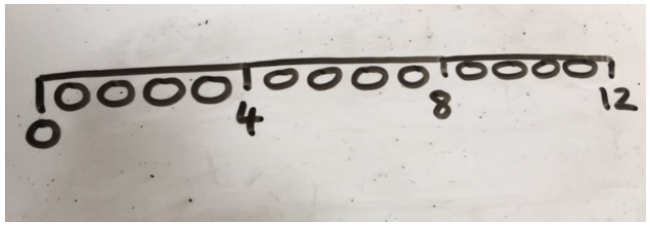
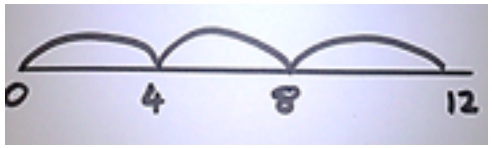
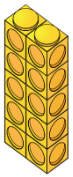
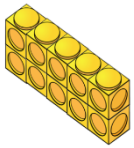
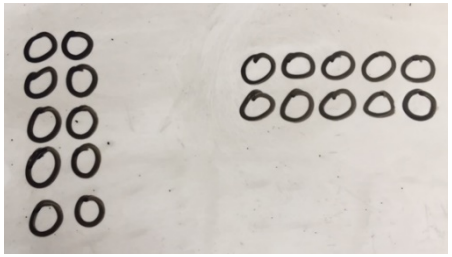
Missing digit calculations

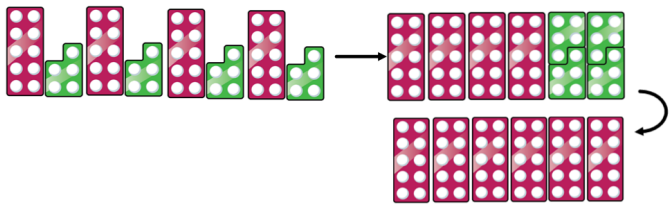
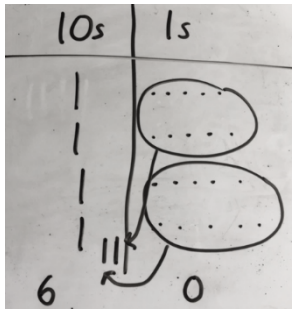
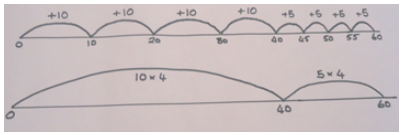
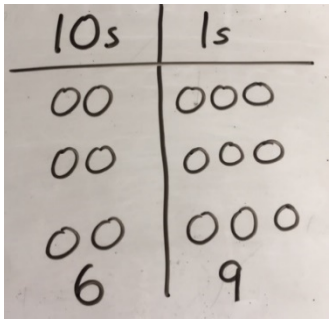
$\begin{array}{r} 3\ 9\ \square \\ -\ \square\ \square\ 6 \\ \hline \square\ 0\ 5 \end{array}$

Calculation policy: multiplication

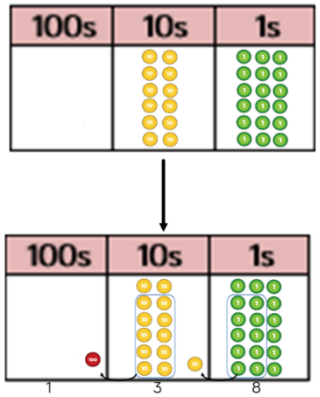
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12

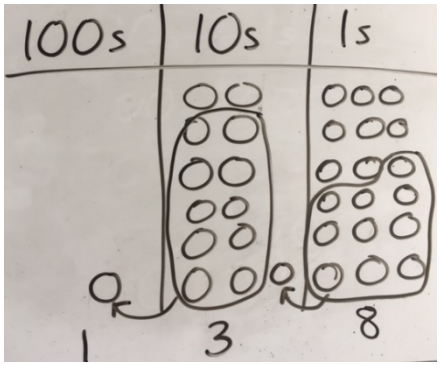
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Children represent this pictorially alongside a number line.  	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5  5 lots of 2	Children represent the arrays pictorially. 	Children use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children represent the concrete manipulatives pictorially, circling to show exchange. 	Children show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters. Base 10 can also be used. 3 × 23 	Children represent the counters pictorially. 	Children record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

Formal column method with place value counters.
6 x 23



Children represent the counters pictorially, circling to show exchange.



Formal method:

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

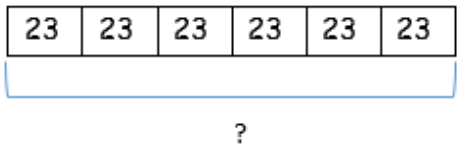
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

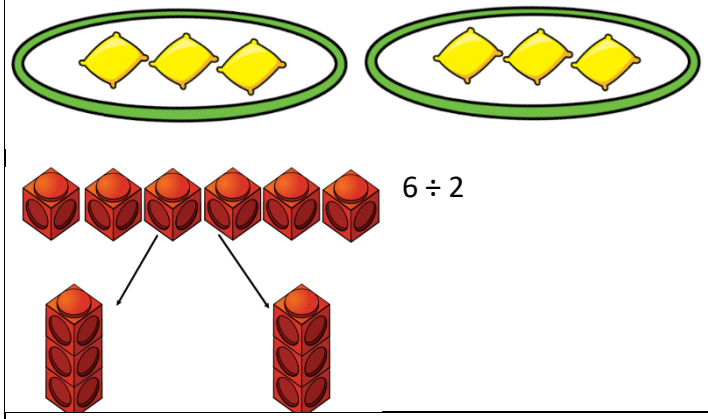
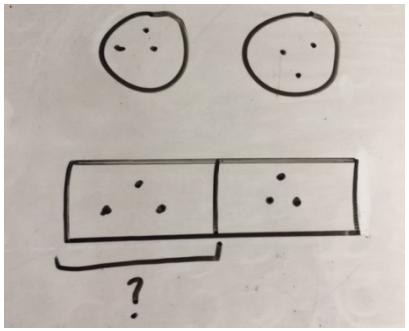
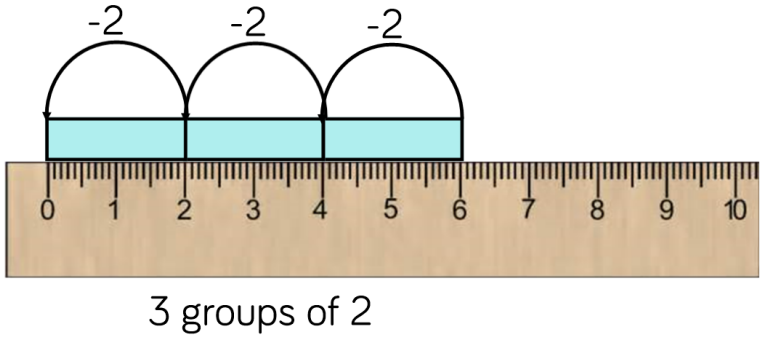
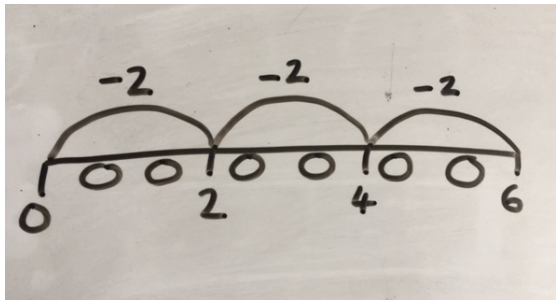
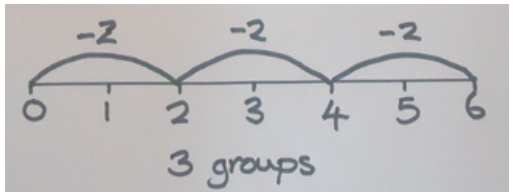
□ = 6 × 23

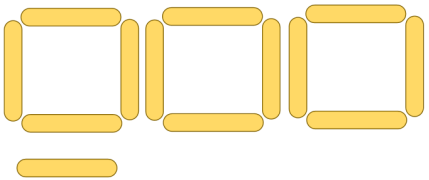
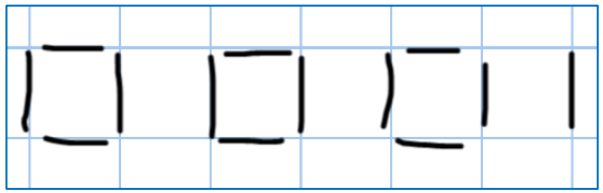
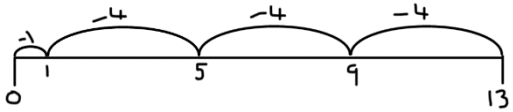
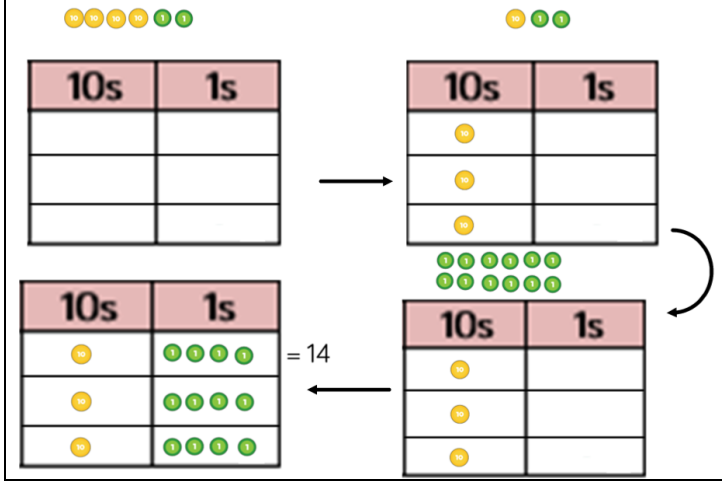
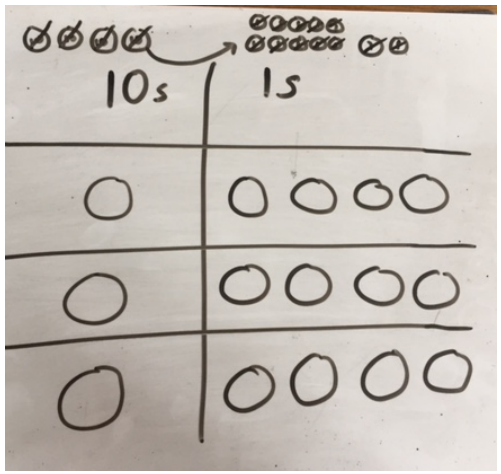
$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

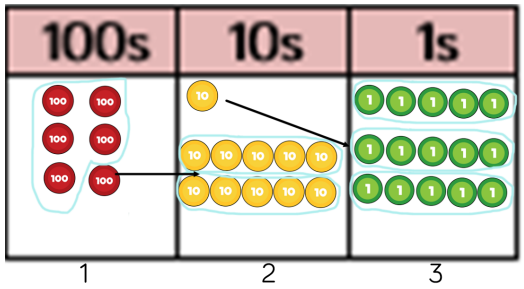
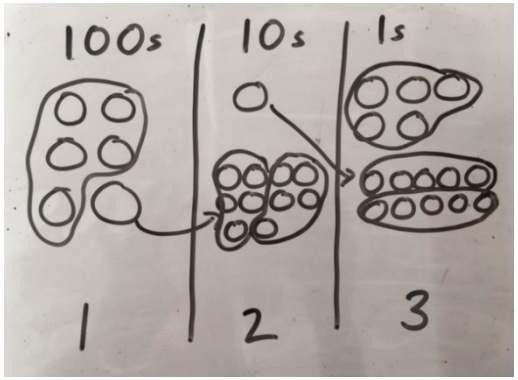
What is the calculation?
What is the product?

Calculation policy: division

Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
Sharing using a range of objects.  6 ÷ 2	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

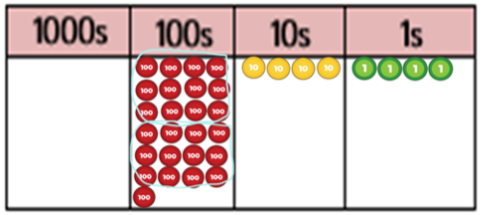
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods and a ruler can also be used (as above). 13 ÷ 4 Use lollipop sticks to form wholes. Squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated subtraction on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children represent the place value counters pictorially, showing exchange. 	Children should write calculations to show they have made sense of the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Formal method: $$\begin{array}{r} 123 \\ 5\overline{)\,{}^{1}6{}^{1}15} \end{array}$$

Long division using place value counters

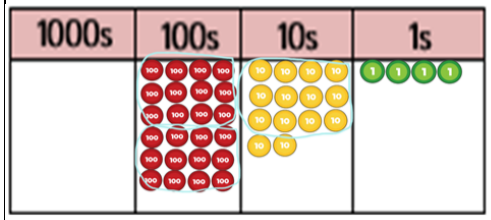
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



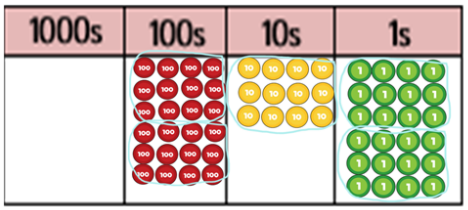
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

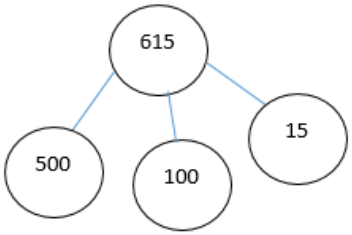
$$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$5\overline{)615}$ 615 ÷ 5 = ☐ = 615 ÷ 5	What is the calculation? What is the answer?